Assignment 4

1. What is the most efficient means of evaluating the polynomial $x^5 + 3x^2 + 4x + 2$?

2. You are introduced to a programming language which has the following features:

   a. assignment is performed with the `:=` operator,
   b. the operators for addition and multiplication are `+` and `*`, respectively,
   c. the language is untyped, so you do not have to declare the type of a local variable, you simply start using it,
   d. a for loop is constructed as follows:
   ```
   for i from m to n do
        # for loop body
   end do
   ```
   e. an array may be indexed from arbitrary initial and final values, including negative indices, and arrays are indexed, like in C++, using square brackets.

Suppose that the coefficients of a polynomial are stored in an array indexed from `0` to `n` and the name of the array is `coeffs`.

3. Evaluate the polynomials

```
-0.2916666666666667*x^4 + 17737.75*x^3 - 404521422.2083333*x^2
                        + 4100171288508.750*x – 15584531083653020.0
```

and

```
-0.2916666666666667*x^4 - 0.25*x^3 + 2.791666666666667*x^2
                        + 1.75*x + 4.0
```

at the values $x = 15203.6$ and $x = –0.4$, respectively. The first is a polynomial interpolating the points

(15202, 9), (15203, 5), (15204, 4), (15205, 8), (15206, 12)

and the second interpolates

(–2, 9), (–1, 5), (0, 4), (1, 8), (2, 12).

Should these two evaluate the same result? Which calculation is likely more accurate?

4. Explain how you would go around finding and evaluating the polynomial that interpolates the four points (15.2, 0.5), (15.3, 0.7), (15.4, 0.4), (15.5, 0.2) at the point $x = 15.32$? What would be the Vandermonde matrix? What is the system of linear equations you are solving? At what point do you evaluate the interpolating polynomial at? You do not have to solve any system of linear equations, but you can indicate what you would do at each step under the assumption you have performed the previous calculation.

5. Explain how you would go around finding and evaluating the polynomial that interpolates the three points (15.2, 0.5), (15.3, 0.7), (15.4, 0.4) at the point $x = 15.38$ assuming you do not have access to information at $x = 15.5$. What would be the Vandermonde matrix? What is the system of linear equations you are solving? At what point do you evaluate the interpolating polynomial at? You do not have to solve any system of linear equations, but you can indicate what you would do at each step under the assumption you have performed the previous calculation.

6. Approximate the derivative and second derivative at the point $x = 3.2$ given the three points (3.1, 4.7), (3.2, 4.9) and (3.3, 5.0).

7. Approximate the derivative and second derivative at the point $x = 3.3$ given the four points (3.0, 4.4), (3.1, 4.7), (3.2, 4.9) and (3.3, 5.0) so that the result is $O(h^2)$.

8. Calculate the second derivative of $\sin(x)$ at the point $x = 7.3$ using the points $x = 7.1$, 7.2, and 7.3, and then again using the points $x = 7.2$, 7.3, and 7.4. Which has the larger error, and why?